tend to introduce greater or less errors. For example, the flow in the second medium must have been somewhat disturbed by the circumstance that the non-conducting walls of the intermediate cell were parallel to the direction of the incident, not refracted flow, which may possibly have disturbed the course even as far as the middle of the cell. Again, there would have been a much larger resistance in the porous wall than in a stratum of equal thickness of one of the electrolytes, and the thickness or porosity of the wall, or the proportion of the two electrolytes imbibed by it, may have varied somewhat in a lateral direction. The numbers obtained cannot therefore be deemed sufficient to decide between two such laws as that of sines and that of tangents. In case of the second medium being the better conductor, it is evident that the law of sines would lead to extravagant results, as there can be no such thing as total internal reflection. The alteration in the direction of the equipotential surfaces, and, therefore, of the lines of flow, in passing from one metal into another of different conducting power, has already been investigated experimentally by Quincke, and the results of experiment compared with theory. (" Pogg. Ann.," vol. xcvii [1856], p. 382.)—G. G. S., 14 June, 1881.]

## XX. "Note on the Spectrum of Sodium." By Captain W. de W. Abney, R.E., F.R.S. Received June 14, 1881.

On examining the spectra of different metals, there is one point which is striking in the extreme, viz., the absence of any very marked lines in the region between $\lambda$ 7000 and $\lambda$ 7600, which latter number we may take as the visible limit of the spectrum. With the exception of the well-known pair of lines of potassium, I am not aware that any lines in metallic spectra, which have been carefully studied, have been found below this limit, though recently, in the spectra of some of the rarer earths, I believe some few lines have been recorded.

Having photographed the emission spectra produced in the arc of several metals, it appears, so far as examination has been made, that only those which can be volatilised at a low temperature have any lines in the infra-red region. Sodium is an example of this. It has a pair of lines at wave-length of about 8187 and 8199 of an intensity of about 3, taking the intensity of D lines as 10. It will be noted that the difference in wave-length between this pair is greater than that of the D lines. They do not seem to have any corresponding dark lines in the solar spectrum, though there are three faint lines which lie close to these wave-lengths.

In the calcium spectrum there is a pair of very faint lines which lie between $\lambda$ 8500 and 8600. Their exact wave-lengths have not at present been determined.

Iron, cobalt, nickel, copper, magnesium, and potassium have, up to the present, given negative results, but will be examined again.

## XXI. "Formulæ for $\operatorname{sn} 8u$, $\operatorname{cn} 8u$, $\operatorname{dn} 8u$, in terms of $\operatorname{sn} u$." By Ernest H. Glaisher, B.A., Trinity College, Cambridge. Communicated by J. W. L. Glaisher, M.A., F.R.S. Received June 16, 1881.

(Abstract.)

In Grunert's "Archiv der Mathematik und Physik," vol. xxxvi (1861), pp. 125–176, Baehr has given the formulæ for $\operatorname{sn} nu$, $\operatorname{cn} nu$, $\operatorname{dn} nu$ in terms of $\operatorname{sn} u$ for the cases $n=2$, 3, 4, 5, 6, 7. These expressions are reproduced in a tabular form in Cayley's "Treatise on Elliptic Functions," Art. 109.

The present paper contains the corresponding formulæ for the case of $n=8$. Denoting the numerators and common denominator of $\operatorname{sn} 4u$, $\operatorname{cn} 4u$, $\operatorname{dn} 4u$ by P, Q, R, S respectively, then the numerators and common denominator of $\operatorname{sn} 8u$, $\operatorname{cn} 8u$, $\operatorname{dn} 8u$ are respectively $2PQRS$, $S^4-2P^2S^2+k^2P^4$, $S^4-2k^2P^2S^2+k^2P^4$, $S^4-k^2P^4$; and the paper contains the values of these quantities and also of $P^2$, $S^2$, $P^4$, $S^4$ in terms of $\operatorname{sn} u$, arranged in a tabular form.

## XXII. "On Riccati's Equation and its Transformations, and on some Definite Integrals which satisfy them." By J. W. L. Glaisher, M.A., F.R.S., Fellow of Trinity College, Cambridge. Received June 16, 1881.

(Abstract.)

The memoir relates chiefly to the different forms of the particular integrals of the differential equation

$$\frac{d^2u}{dx^2}-a^2u=\frac{p(p+1)}{x^2}u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

and to the evaluation of certain definite integrals which are connected with this equation.

Transforming (1) by assuming $u=x^{-p}v$ and putting $2p=n-1$, it becomes

$$\frac{d^2v}{dx^2}-\frac{n-1}{x}\frac{dv}{dx}-a^2v=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

and this may be transformed into Riccati's equation,